Test 2 **MATH 125- Section 7** **October 19, 2017**

Name: _____ **Signature:** _____

Show all your work!

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1. [25 points] Find the following indicated derivatives

   (a) $f'(w)$, where $f(w) = 2^{-w} + \frac{1}{w} - \ln w$.

   (b) $h'(z)$, where $h(z) = \tan(e^{\pi}) - \arctan(a\sqrt{z})$.

   (c) $g'(\theta)$, where $g(\theta) = \sin(4\theta) + \tan^3(3\theta^2)$.

   (d) $v'(t)$, where $v(t) = \sqrt{t + \sin^2(2t+1)}$.

   (e) $K'(x)$, where $K(x) = 3^{2x} + (2x)^3$.

2. [10 points] Find the x values at which the following function has a horizontal tangent line

$$f(x) = \frac{x}{1+x^2}.$$

3. [20 points] Find the equation of the tangent line of the curve

$$\ln(xy) = 2x$$

at the point $(1, e^2)$.

4. [25 points] Let $f(x) = xe^{-2x}$.

   (a) Over what intervals f is increasing? decreasing?

   (b) Find the local maxima and local minima of f.

   (c) Over what intervals f is concave up? concave down?

5. [10 points] Given that $f(5) = 13$ and $f'(5) = -0.5$,

   (a) use local linearization to approximate $f(7.1)$

   (b) Suppose that $f''(x) < 0$ for all x, is the approximation in part (a) an underestimate or an overestimate? Explain.

6. [20 points] Given that $f'(4) = 10$ and $f^{-1}(0) = 4$, find

   (a) the equation of the tangent line of $g(x) = xf(x)$ at $x = 4$.

   (b) the equation of the tangent line of f^{-1} at $x = 0$.

7. [20 points] A company manufactures cylindrical barrels with no tops. The bottom of the barrels are to be made with material that costs \$8 per square foot and the rest is made with a material that costs \$6 per square foot. If each barrel is to hold 10 cubic feet, find the dimensions of the barrel that will minimize the total cost.

8. [20 points] The graph of the differentiable function f lies entirely above the x-axis, suppose that $f'(x) < 0$ for all x.

   (a) Find the points at which $g = 1/f$ has absolute maximum and absolute minimum in the interval $[1, 5]$.

   (b) For the function $h(x) = f(x^2)$, find the following

   - all the critical points.
   - intervals at which h is increasing or decreasing.
   - the points at which h has local maximum or local minimum.